Study on dynamical critical exponents of the Ising model using the damage spreading method

1995 J. Phys. A: Math. Gen. 28 4543

(http://iopscience.iop.org/0305-4470/28/16/012)

J. Phys. A: Math. Gen. 28 (1995) 4543–4552. Printed in the UK

Study on dynamical critical exponents of the Ising model using the damage spreading method

Fugao Wang†, Naomichi Hatano and Masuo Suzuki
Department of Physics, Faculty of Science, University of Tokyo, Tokyo 113, Japan

Received 21 April 1995

Abstract. The dynamical behaviour of the Ising model with heat-bath dynamics is studied. According to our investigation of the model using the damage spreading method, the dynamical phase transition temperature is estimated as 4.52 ± 0.03 for the 3D Ising model. The dynamical critical exponent z has been obtained as 2.16 ± 0.04 for the 2D ferromagnet and 2.09 ± 0.04 for the 3D Ising model. The scaling property of magnetization is obtained.

1. Introduction

In recent years, more and more attention has been paid to the study of the dynamical phase transition and critical exponents. A lot of simulation research on large-size spin system has been carried out to estimate the precise values, such as the critical temperature and dynamical critical exponent z. But with different (analysis or simulation) methods, the value for z varies over a relatively large range from 2.00 to 2.30. It is even argued that the value of the dynamical critical exponent z is uncertain. For the two-dimensional Ising model, with the dynamical high-temperature expansion method, Yahata and Suzuki [1] first found that the dynamical exponent $z = 2.00 \pm 0.05 > \gamma$, which was previously believed to be $z = \gamma$ according to the conventional van Hove theory. The real-space RG method gave a value of 2.23 [2], and the CAM approach reported the result as 2.15 ± 0.02 [3, 4]. Meanwhile Poole and Jan [5] and Manna's [6] simulation showed values as high as 2.24 and 2.27, but MacIsaac and Jan [7] and Ito's [8] Monte Carlo simulations came to the conclusion that $z = 2.16 \pm 0.02$. Recent simulations on large systems seem to support the value with $z = 2.165 \pm 0.010$ [8]. For the three-dimensional Ising model, more inconsistent results have been obtained with different approaches. The renormalization-group estimation is $z \simeq 2.02$. With multispin coding algorithms, Wansleben and Landau [9, 10] have calculated the dynamical exponent $z = 2.04 \pm 0.03$, which is consistent with the RG result. By simulation studies on the properties of magnetization and energy, Heuer [11] has obtained a large value as $z = 2.10 \pm 0.02$, and Stauffer [12] and Ito's [8] simulation on large systems suggested $z = 2.06 \pm 0.02$. Very recently Matz *et al* [13] applied MacIsaac's simulation method from the two-dimensional Ising model to the three-dimensional one, they reported a much larger value $z = 2.35 \pm 0.05$ by the normal scale ($\tau \propto L^z$), but when the scale was taken as $\tau \propto L^z d_f \ln(L)$, they got the value $z = 2.05 \pm 0.05$, which was much closer to the 'consensus' value [14].

† Permanent address: Department of Applied Physics, Shanghai Jiaotong University, Shanghai 200030, People's Republic of China.

0305-4470/95/164543+10$19.50

About the critical temperature, there are also some arguments. Very recently, Grassberger [15] argued that the damage spreading critical temperature (T_d) was not the same as that of its corresponding static model (T_S). With Glauber dynamics, he concluded that $T_d/T_S \simeq 0.992$ for the 2D Ising model and $T_d/T_S \simeq 0.92$ for the 3D Ising model. According to the above discussion, the dynamical critical temperature and the precise value of z are still very open questions [14].

In this paper, we will study the dynamical phase transition of the Ising model with nearest-neighbour pair interactions by the damage spreading method first. Then the dynamical critical exponent z is estimated at the critical temperature. In section 2, the dynamical Ising model and damage spreading method are introduced. In section 3, the curve of damage distance is investigated for the 3D dynamical Ising model at a finite Monte Carlo step. The transition temperature is estimated by the damage spreading method in this section. We will also calculate the dynamical critical temperature of the 3D Ising model with a finite scaling method. In section 4, the dynamical exponent z is estimated by three different methods. We will estimate the exponent z directly by calculating the average damage vanishing time for two different initial spin configurations. We will also estimate the exponent z by considering the short-time relaxation and integral scaling property of magnetization at the dynamical phase transition temperature. With our new dynamical critical exponent, the scaling property of the magnetization is given in section 5. Conclusions are presented in section 6.

2. The dynamical Ising model and the damage spreading method

In the damage spreading method [15–25], the damage distance between two different initial spin configurations is measured as they evolve in the same thermal noise. For a large class of spin models in statistical mechanics, the sharp dynamical phase transitions are observed with this method and these dynamical phase transition points separate two phases. One is a high-temperature (disorder) phase where spin distance is independent of the initial configurations and vanishes very rapidly. The other is a low-temperature (order) phase where the distance of two configurations remains finite for a long time. According to Derrida's investigations on the 3D spin-glass model [19] and the two-dimensional classical XY model [20], the third phase between these two phases was found. It is called the intermediate phase, where the distance does not vanish but becomes independent of the initial condition. The damage spreading method is especially important in the cases, such as the spin-glass problems and commensurate–incommensurate transitions, where the equilibrium phase transitions are hard to detect with standard Monte Carlo simulations. Recently, by studying damage spreading of spin models in a temperature gradient, Batrouni and Hansen [21] have obtained not only the critical temperatures but also critical exponents with high precision, such as the correlation length exponent ν and the second critical exponent β. Glotzer *et al* [22] have also determined the static thermodynamic quantities correctly from damage spreading.

Although it is a dynamical method, it has been argued that the transition points found with this approach often reflect the corresponding equilibrium transitions. Golinelli and Derrida [20] have applied this method to the two-dimensional classical XY model. They have observed three regimes as in the case of the 3D spin glass [23, 24]. One of their dynamical phase transition points ($T_2 = 0.95 \pm 0.05$) coincides with the corresponding equilibrium transition point ($T_{KT} = 0.9$) predicted by Kosterlitz and Thouless. Chiu and Teitel [25] have studied the same model with different dynamics and distance which preserve the rotational invariance of the Hamiltonian. Their results contradict Golinelli and Derrida's results. According to their investigations, only low- and high-temperature phases have been found and the transition occurs close to the Kosterlitz–Thouless transition $T_C = 0.89$. From these investigations, it is suggested that damage spreading transitions may be related to the corresponding equilibrium transitions. Now, it is generally believed that for the dynamical Ising model the dynamical phase transition occurs at the same temperature as its corresponding static model.

In this paper, we study the Ising models on the 2D square and 3D cube lattice under periodic boundary condition. The linear lattice size is L. For a given configuration $\{\sigma_i(t)\}$, the Hamiltonian can be written as

$$-\beta\mathcal{H}(t) = \sum_{\langle ij\rangle} J\sigma_i(t)\sigma_j(t) \tag{1}$$

where Σ sums over all nearest-neighbour pairs of spins and J is the exchange interaction coefficient. To make the configuration $\{\sigma_i(t)\}$ evolve in time, we apply the heat-bath dynamics. From the configuration $\{\sigma_i(t)\}$ at time t, one chooses the site $\sigma_i(t)$ at the time $t+1$ by

$$\sigma_i(t+1) = \begin{cases} 1 \text{ with probability } \frac{1}{2} + \tanh\left[\sum_k J\sigma_k(t)\right]/2 \\ -1 \text{ with probability } \frac{1}{2} - \tanh\left[\sum_k J\sigma_k(t)\right]/2 \end{cases} \tag{2}$$

where $\sum_k$ is over all nearest spins of $\sigma_i(t)$. The temperature is defined by $T = J^{-1}$. To implement this dynamics, we choose a random number $0 \leqslant Z_i(t) \leqslant 1$ for each site and obtain

$$\sigma_i(t+1) = \text{sign}\left\{\frac{1}{2} + \tanh\left[\sum_k J\sigma_k(t)\right]/2 - Z_i(t)\right\}. \tag{3}$$

This means that if the transition probability $W = \frac{1}{2} + \tanh[\sum_k J\sigma(t)]/2 > Z_i(t)$ then $\sigma_i(t+1) = +1$, otherwise $\sigma_i(t+1) = -1$.

To compare the time evolution of two configurations $\{\sigma_i(t)\}$ and $\{\bar{\sigma}_i(t)\}$ subjected to the same thermal noise. At each time step, the same random number $Z_i(t)$ is chosen for the ith site of the two configurations $\{\sigma_i(t)\}$ and $\{\bar{\sigma}_i(t)\}$:

$$\begin{aligned} \sigma_i(t+1) &= \text{sign}\left\{\frac{1}{2} + \tanh\left[\sum_k J\sigma_k(t)\right]/2 - Z_i(t)\right\} \\ \bar{\sigma}_i(t+1) &= \text{sign}\left\{\frac{1}{2} + \tanh\left[\sum_k J\bar{\sigma}_k(t)\right]/2 - Z_i(t)\right\}. \end{aligned} \tag{4}$$

Two configurations have different initial conditions. If they meet at a finite time t ($\{\sigma_i(t)\} = \{\bar{\sigma}_i(t)\}$), it is not difficult to find that two configurations remain identical after that time t. So the distance between $\{\sigma_i(t)\}$ and $\{\bar{\sigma}_i(t)\}$ can be defined as

$$D(L,T,t) = \sum_i \left|\sigma_i(t) - \bar{\sigma}_i(t)\right|/2N \tag{5}$$

where $N = L^d$ is the number of sites of the spin system. We use the distance to measure the correlation between two sets of spin configurations. In general, $D(L,T,t)$ depends on the temperature, the simulation time t, the system size L the initial conditions $\{\sigma_i(0)\}$ and $\{\bar{\sigma}_i(0)\}$, dynamics, the thermal noise and the boundary conditions.

In calculation, we average the distances over many samples. The average distance of the two spin configurations is defined as

$$\langle D(L,T,t)\rangle = \frac{1}{N_S}\sum_{j=1}^{N_S} D_j(L,T,t) \tag{6}$$

where $D_j(L,T,t)$ is the damage distance for the jth independent trial, N_S is the number of independent samples, and the sum is over all trials.

3. Dynamical phase transition of the 2D and 3D Ising models

For the 2D ferromagnet, Newmann and Derride [27] have applied the damage spreading method to get the results as $T_C = 2.25 \pm 0.05$, which is consistent with the corresponding static phase transition. For the 3D Ising model, we use the same method to demonstrate the dynamical phase transition. The dynamical transition temperature is also estimated by the finite scaling theory.

The simulation result of $\langle D(L,T,t)\rangle$ is shown in figure 1 as a function of the temperature T at a finite Monte Carlo step with different lattice sizes. Two kinds of initial conditions are chosen:

(i) $\{\sigma_i(0)\}$ is random and $\bar{\sigma}_i(0) = -\sigma_i(0)$, so $\langle D(L,T,0)\rangle = 1$
(ii) $\{\sigma_i(0)\}$ and $\{\bar{\sigma}_i(0)\}$, are random and independent, so $\langle D(L,T,0)\rangle = \frac{1}{2}$.

For different L, we have simulated the average damage distance at $t = 500$ MCS over $N_S = 100$ samples for different temperatures and find two temperature regions as shown in figure 1. At the high temperature, the distance vanishes or decreases very rapidly subject to thermal noise. At a time before $t = 500$ MCS, two configurations become identical and since then the distances between two configurations remain zero. The system in this temperature range is under disorder phase. Because each spin configuration change rapidly with thermal noise at high temperature, the spontaneous magnetization of each configuration $m_S(t)$,

$$m_S(t) = \langle\sigma_i(t)\rangle = \langle\bar{\sigma}_i(t)\rangle = 0\,. \tag{7}$$

By comparing with the ferromagnetic–paramagnetic transition, the spin system can also be assumed to be in the paramagnetic phase at high temperature.

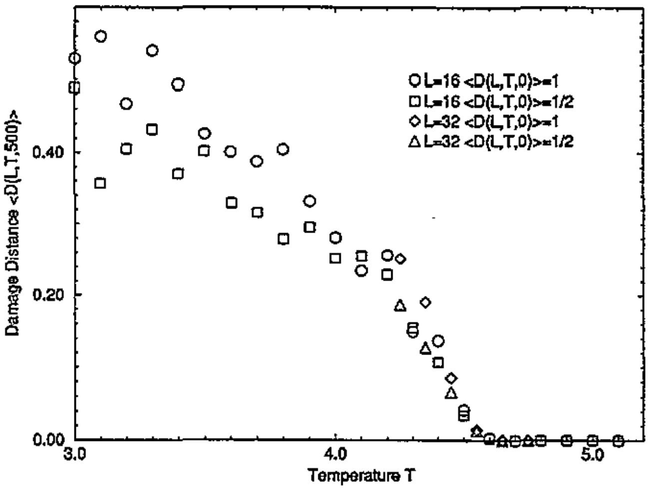

Figure 1. The average distance at $t = 500$ MCS against the temperature. The distance is obtained by the average value over 100 samples. Two phases are observed.

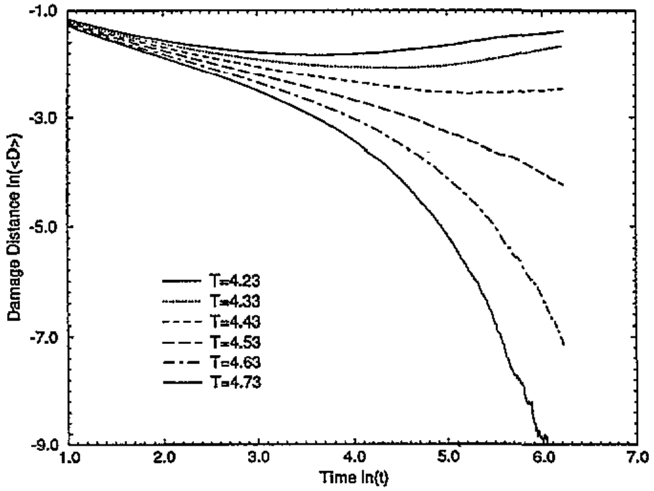

Figure 2. The time decay of the average damage distance at different temperatures for $L = 32$. The transition temperature is 4.48 ± 0.05.

At low temperature $\langle D(L, T, t)\rangle$ remains finite and depends on its initial condition. The reason is that the spins of the Ising model flip slowly subject to the thermal noise. Even at $t = 500$ MCS per site, two configurations are different and the distance between two configurations defined as (5) and (6) does not vanish. So the spin system is under order phase. If we choose the initial spin configurations as $\{\sigma_i(0)\} = -\{\bar{\sigma}_i(0)\} = 1$, then at low temperature, the spontaneous magnetization

$$m_S(t) = \langle\sigma_i(t)\rangle \neq 0 . \tag{8}$$

Similarly, the spin system is under ferromagnetic phase. This situation is quite similar to the equilibrium phase transition of the ferromagnetic Ising model. Figure 1 shows the dynamical transition from the disorder to order phase when the system cools down. From figure 1, the transition temperature is difficult to estimate precisely. By considering the time decay of two spin configurations' damage distance at different temperatures, the dynamical transition temperature can be estimated. The figure of average damage versus time for different temperatures is shown in figure 2. When $T < 4.43$, the average distance does not vanish before 500 MCS. But when the system temperature is above 4.53, the damage of the two configuration decreases steeply. So the transition temperature can be estimated from the figure 2 as $T_C = 4.48 \pm 0.05$.

The more precise value of critical temperature is estimated by the finite scaling theory. With definition of the measure characteristic time τ_1 and characteristic square time τ_2 as

$$\begin{aligned} \tau_1(L, T) &= \frac{\sum_t t\langle D(L, T, t)\rangle}{\sum_t \langle D(L, T, t)\rangle} \\ \tau_2(L, T) &= \frac{\sum_t t^2\langle D(L, T, t)\rangle}{\sum_t \langle D(L, T, t)\rangle} \end{aligned} \tag{9}$$

and scaling forms:

$$\begin{aligned} \tau_1(L, T) &= u(L) f_1(v(L)(T - T_C)) \\ \tau_2(L, T) &= u^2(L) f_2(v(L)(T - T_C)) . \end{aligned} \tag{10}$$

We can find that the ratio

$$R(L, T) = \frac{\tau_2(L, T)}{\tau_1^2(L, T)} = f_3(v(L)(T - T_C)) \tag{11}$$

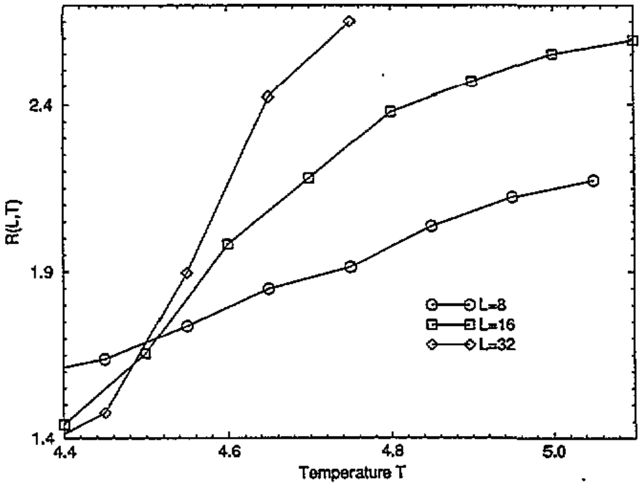

Figure 3. The ratio $R(L,T)$ versus temperature T for the 3D Ising model. All curves cross at $T_C = 4.52 \pm 0.03$.

is independent of lattice sizes at dynamical critical temperature T_C. This means that for different L, all curves $\langle R(L,T)\rangle$ plotted as functions of T should cross at the same temperature, i.e. the critical temperature T_C. The curves are shown in figure 3 for $L = 8, 16$ and 32, from which we get the critical temperature as $T_C = 4.52 \pm 0.03$. The recent simulation result for T_C of the 3D Ising model is about 4.5115 [14]. From our result for the 3D dynamical Ising model, the dynamical critical temperature is also consistant with its corresponding equilibrium critical temperature.

With heat-bath dynamics, the dynamical phase transition in this model occurs near its corresponding equilibrium transition, which is similar to Golinelli and Derrida [20] and Chiu's [25] work. But our result contradicts Grassberger' conclusion on the 3D dynamical Ising model with Glauber dynamics [15]. In our research, the distance of two spin configurations always depends on the initial distance before they vanish. This is also different from the spin glass [19] and XY model [20], in which the intermediate-temperature phase was found.

In the following sections, we choose the critical temperature T_C as $2/\ln(1+\sqrt{2})$ for the 2D and 4.5115 for the 3D Ising models.

4. Dynamical critical exponent

4.1. By considering the average vanishing time

Normally, the dynamical critical exponent z is defined at the dynamical critical temperature as $\tau \sim L^z$, where L is the linear lattice size and τ is the relaxation time for the dynamics. In the damage spreading method, the relaxation time can be defined by the time for damage to vanish. Near the critical temperature, the fluctuation is very strong. To measure the precise vanishing time from finite samples is difficult, because one may expect the time to be in a very large range for different samples. This is also one of the reasons for the uncertainty of the exponent z. As we know, average distance $\langle D(L,T,t)\rangle$ actually is the ratio of damaged sites, which means the survival probability for a site to be damaged at t MCS. In this paper, we use the characteristic time as

$$\tau(L,T) = \frac{\sum_t t\langle D(L,T,t)\rangle}{\sum_t \langle D(L,T,t)\rangle} \tag{12}$$

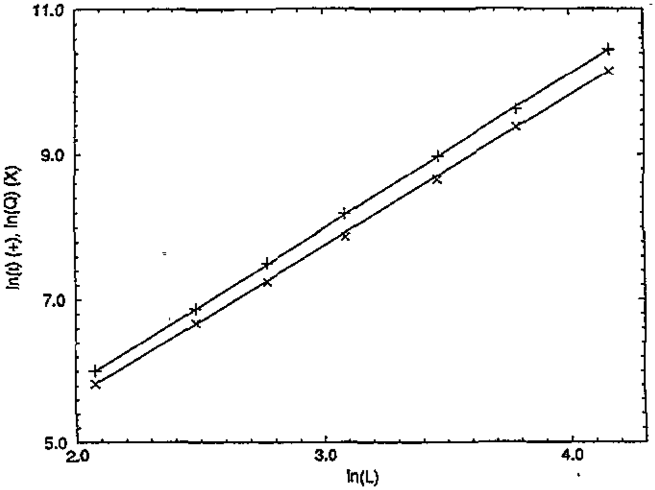

Figure 4. The log–log plots for the vanishing time τ (+) and value $Q(L)$ (×) against the linear lattice size L for the 2D Ising model. The dynamical exponents are estimated by the slopes of the lines as $z = 2.12$ and 2.16.

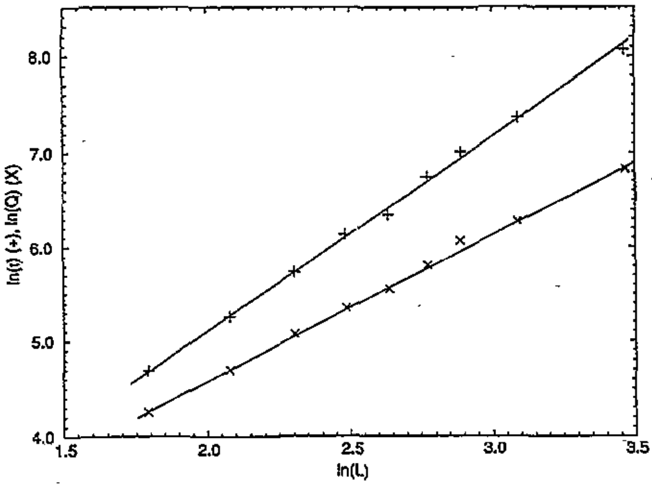

Figure 5. Similar plots as figure 4 for the 3D Ising model, the estimations for z are about 2.05 and 2.09.

to measure the vanishing time. We choose the following all initial damaged configurations:

$$\{\sigma_i(0)\} = 1 = -\{\bar{\sigma}_i(0)\}\,. \tag{13}$$

We estimate the vanishing time at the dynamical critical temperature for this initial condition and repeat the simulation on different lattice sizes L for the 2D Ising model. In figure 4, the vanishing time for different L is denoted by '+' for the 2D Ising model. For every point in the figure, at least 10^{10} single spin flips have been flipped. From the slope of the line the dynamical exponent is estimated as $z \simeq 2.12$. Similar research has been carried out on the 3D Ising model and the result is shown in figure 5 with $z \simeq 2.05$.

4.2. By considering the magnetization relaxation property for short time on large-size lattice

When the initial condition is chosen as $\{\sigma_i(0) = 1\}$ and $\{\bar{\sigma}_i(0)\} = -1$ [13], $\langle D(L, T, t)\rangle$ is equivalent to magnetization in this case. That is,

$$\langle D(L, T, t)\rangle = \frac{\langle \sigma_i(t) - \bar{\sigma}_i(t)\rangle}{2} \equiv m(L, T, t)\,. \tag{14}$$

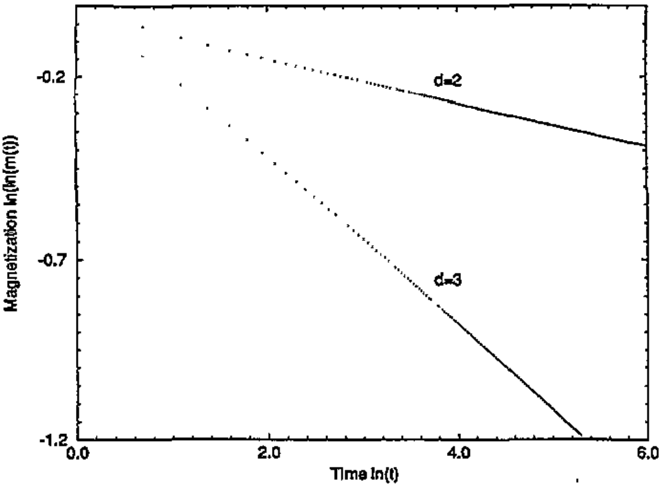

Figure 6. The magnetization relaxation value on $L = 400$ for the 2D and $L = 64$ for the 3D Ising model. Between $t = 100$ and $t = 200$ MCS, the slopes of the lines show about 0.0571 and 0.243, which mean $z \simeq 2.19$ for the 2D and 2.13 for the 3D Ising model.

By considering scaling theory, the time-dependent magnetization $m(L, T, t)$ from all up-spin configuration behaviour near the critical points as [28, 29]

$$m(T, L, t) \sim L^{-\beta/\nu} f(L^{1/\nu}(T - T_C), tL^{-z}) \tag{15}$$

where the static exponent $\beta/\nu = \frac{1}{8}$ for the 2D and 0.518 ± 0.007 for the 3D Ising model [14]. At $L \to \infty$, and $T = T_C$, for $tL^{-2z} \ll 1$, from the above scaling form, it is easy to conclude that the magnetization behaves as [29]

$$m(t) \equiv m(T_C, \infty, t) \sim t^{-\beta/z\nu} . \tag{16}$$

By simulating on a large lattice with short Monte Carlo steps, we can estimate the value of z with the slope of the line in figure 6 for the 2D Ising model. For the 2D Ising model when the linear size $L > 60$, the lines for different lattice sizes can hardly be distinguished. We estimated the lattice as large as $L = 400$ to determine z. We got $z \simeq 2.19$ for the two-dimensional case. With the same method, we obtained $z \simeq 2.13$ for the three-dimensional Ising model ($L = 64$) as figure 6 shows. For each line, about 10^{10} single spin flips have been done.

4.3. By considering the magnetization integral scaling property for long-time simulation on relatively small size lattice

From the general scaling formula (15), we can calculate the integral

$$Q(L) \equiv \int_0^\infty m(T_C, L, t)\mathrm{d}t = \sum_{t=1}^{\infty} m(T_C, L, t) \sim L^{-\beta/\nu+z} . \tag{17}$$

From this simple scaling relation and static critical exponent, the dynamical exponent can be estimated easily. The points denoted with '×' in figure 4 show the simulation results $Q(L)$ for the different lattice size L of the two-dimensional Ising model. The exponent z can be determined by the slope of the line as about 2.16. A similar investigation of the 3D Ising model shows $z \simeq 2.09$ in figure 5.

The merit of this method is that it considers not only the short-time but also the long-time relaxation properties.

4.4. *Our final result for the dynamical exponents z*

We have estimated the dynamical exponent z by three methods. In the first method, only the long vanishing time is considered. In the second method, we have obtained z by studying the short-time relaxation property for magnetization. In the last method, we considered both of the above features by introducing an integral and obtained the exponent between the results by of the above two methods. With the above three different estimations for z, we can conclude $z = 2.16 \pm 0.04$ for the 2D and $z = 2.09 \pm 0.04$ for the 3D Ising model with the heat-bath dynamics.

5. The scaling property of magnetization

With the new data for the dynamical critical exponent, we investigate the scaling property of magnetization at the dynamical critical temperature. In figure 7 we have given the curves for scaling quantities of magnetization and time. We find that all curves satisfy the scaling formula (15) very well in a wide region for the 2D and 3D Ising models.

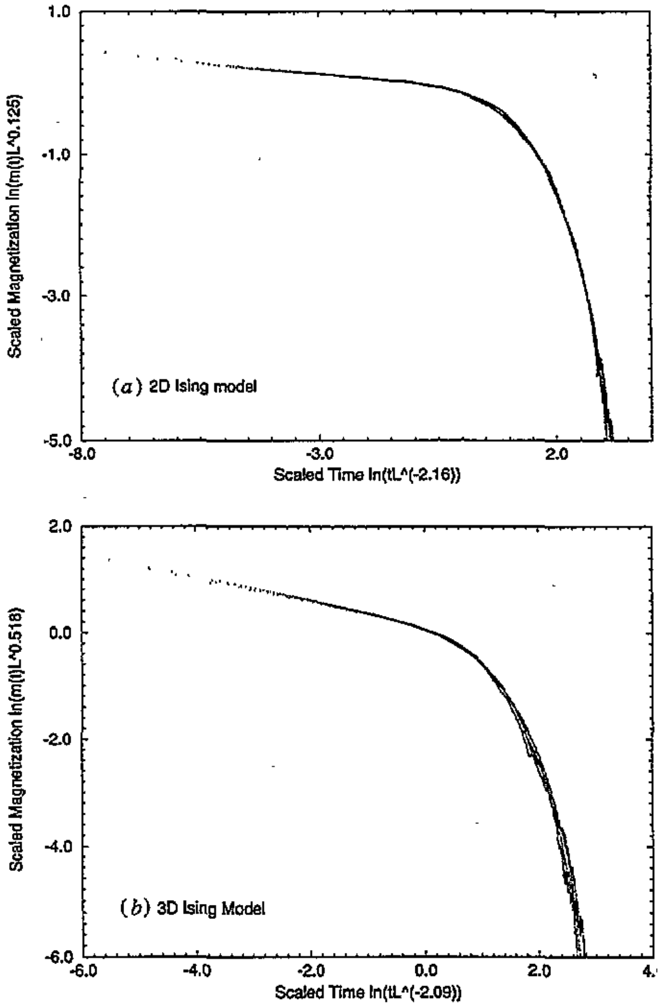

Figure 7. The scaling property of magnetization for (a) the 2D Ising model with $L = 8$, 12, 16, 32 and (b) the 3D Ising model with $L = 6$, 8, 10, 14.

6. Conclusion

With the damage spreading simulation method and finite scaling theory, we have obtained the dynamical phase transition $T_C = 4.52\pm0.03$ for the 3D Ising model, which is consistent with its corresponding static critical temperature. We also obtain the dynamic critical exponent $z = 2.16 \pm 0.04$ for the 2D Ising model and $z = 2.09 \pm 0.04$ for the 3D Ising model. Our dynamical exponent z for the 2D Ising model seems to support MacIsaac and Jan [7] and Ito's [8] result. For the three-dimensional Ising model, we have obtained a little large value by comparing with the most other simulation results. It is consistant with Heuer's result [11]. Very recently, by investigating the early-time scaling property of the magnetization cumulant, Li *et al* [30] have obtained the dynamical critical exponent as $z = 2.1337\pm0.0041$ for the two-dimensional Ising model. This result seems to support our exponent ($z \simeq 2.12$) obtained by considering the finite-size scaling property of the average vanishing time. With our new exponents, the magnetization satisfies the scaling formula in a very wide region. The scaling property for magnetization has been investigated.

Acknowledgments

One of the present authors (FW) would like to thank the Nisshin–Kazankai Foundation for support. The numerical simulations were performed on the Hewlett Packard Apollo 735 and 700 work stations of the Suzuki research group, Department of Physics, University of Tokyo.

References

[1] Yahata H and Suzuki M 1969 *J. Phys. Soc. Japan* **27** 1421
[2] Takano T and Suzuki M 1982 *Prog. Theor. Phys.* **67** 1332
[3] Katrori M and Suzuki M 1988 *J. Phys. Soc. Japan* **57** 807
[4] Suzuki M 1986 *J. Phys. Soc. Japan* **55** 4205
[5] Poole P and Jan N 1990 *J. Phys. A Math. Gen.* **23** L453
[6] Manna S S 1990 *J. Physique* **51** 1261
[7] MacIsaac K and Jan N 1992 *J. Phys A: Math. Gen.* **25** 3585
[8] Ito N 1993 *Physica* **192A** 604; *Physica* **196** 591
[9] Wansleben B and Landau D P 1991 *Phys. Rev.* B **43** 6006
[10] Wansleben B and Landau D P 1987 *J. Appl. Phys.* **61** 3968
[11] Heuer H O 1992 *J. Phys. A Math. Gen.* **25** L567
[12] Stauffer D 1993 *J. Phys. A: Math. Gen.* **26** L599
[13] Matz R, Hunter D L and Jan N 1993 *J. Stat. Phys.* **74** 903
[14] Landau D P 1994 *Physica* **205A** 41
[15] Grassberger P 1995 *J. Phys. A Math. Gen.* **28** L67
[16] Binder K ed 1992 *Monte Carlo Simulation in Condensed Matter Physics* (Berlin: Springer)
[17] Landau D P ed 1993 *Computer Simulation Studies in Condensed-Matter* vol VI (Berlin: Springer)
[18] Barber M N and Derrida B 1988 *J. Stat. Phys.* **51** 877
[19] Derrida B 1989 *Phys. Rep.* **184** 207
[20] Golinelli O and Derrida B 1989 *J. Phys. A: Math. Gen.* **22** L939
[21] Batrouni G G and Hansen A 1992 *J. Phys. A: Math. Gen.* **25** L1059
[22] Glotzer S C, Poole P H and Jan N 1992 *J. Stat. Phys.* **68** 895
[23] Campbell I A and de Arcangelis L 1991 *Physica* **178A** 29
[24] Derrida B and Weisbuch G 1987 *Europhys. Lett.* **4** 657
[25] Chiu J and Teitel S 1990 *J. Phys. A: Math. Gen.* **23** L891
[26] Coniglio A, de Arcangelis L, Herrmann H J and Jan N 1989 *Europhys. Lett.* **8** 315
[27] Newmann A U and Derrida B 1988 *J. Physique* **49** 1647
[28] Suzuki M 1976 *Phys. Lett.* **58A** 435
[29] Suzuki M 1977 *Prog. Theor. Phys.* **58** 1142
[30] Li Z B, Schulke L and Zheng B 1995 *Phys. Rev. Lett.* **74** 3396